5.3b Definite Integrals and Antiderivatives

Rules for Definite Integrals

$$\int_1^2 x^2\,dx + \int_2^5 x^2\,dx = \int_1^5 x^2\,dx$$

predict how these are related

$$\boxed{\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx}$$

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$$\int_1^2 x^2\,dx = -\int_2^1 x^2\,dx$$

$\frac{7}{3}$ $\quad$ $-\frac{7}{3}$

$$\boxed{\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx}$$

$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^a f(x)\,dx$$

$$= 0$$

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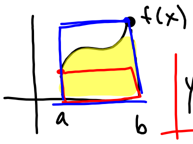
$$\boxed{\int_a^b c\cdot f(x)\,dx = c\int_a^b f(x)\,dx}$$

$$\boxed{\int_a^a f(x)\,dx = 0}$$



$$\boxed{y_{min}(b-a) \le \int_a^b f(x)\,dx \le y_{max}(b-a)}$$

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Average (Mean) Value

$$\bar{y} = f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

↑
height of a rectangle

area of rectangle $= \int_a^b f(x)\,dx$

$f(c)(b-a)$

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Find the average value of f(x) = 4-x^2 on [0,3]. Does *f* actually take on this value at some point on the given interval?

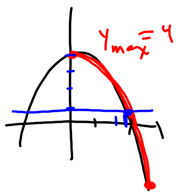
$\bar{y} = \frac{1}{3-0}\int_0^3 4-x^2\,dx$

$\bar{y} = 1 = f(c)$

$= \frac{1}{3}\left(4x - \frac{x^3}{3}\right)\Big|_0^3$

$= \frac{1}{3}\left(4\cdot 3 - \frac{3^3}{3} - 0\right)$

$= 4-3 = 1$



$4 - c^2 = 1$

$c = \sqrt{3} \approx 1.7..$

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Mean Value Theorem for Definite Integrals

If f is continuous on $[a,b]$

then c exists $a \le c \le b$

where $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$

mean value

(average value) $f(c)(b-a) = \int_a^b f(x)\,dx$

rectangle

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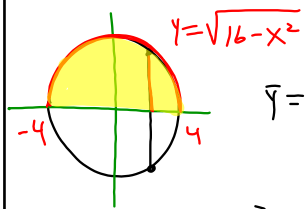
How long is the average chord of a circle of radius 4? Find the value that satisfies the Mean Value Theorem for Definite Integrals.



$\bar{y} = \frac{1}{4--4}\int_{-4}^{4} \sqrt{16-x^2}\,dx$

$= \frac{1}{8}\cdot 8\pi$

circle area $= \pi\cdot 4^2$

$= 16\pi$

$\bar{y} = \pi$

ave chord $= 2\pi$

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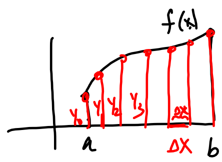

$\bar{y} = \frac{y_0 + y_1 + y_2 + y_3 + \dots + y_n}{n}$

$\Delta x = \frac{b-a}{n}$

$\frac{\Delta x}{b-a} = \frac{1}{n}$

$= \frac{f(x_0) + f(x_1) + \dots + f(x_n)}{n}$

$\sum_{i=0}^{n} f(x_i)\cdot\frac{1}{n}$

$\sum_{i=0}^{n} f(x_i)\cdot\frac{\Delta x}{b-a}$

$\lim_{n\to\infty}\frac{1}{b-a}\sum_{i=0}^{n} f(x_i)\,\Delta x$

$\bar{y} = \frac{1}{b-a}\int_a^b f(x)\,dx$

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